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D6+D0 and five dimensional spinning black hole

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Abstract: We study the system of D6+D0 branes at sub-stringy scale. We show that the proper description of the system, for large background field associated with the D0-branes, is via spinning chargeless black holes in five dimensions. The repulsive force between the D6-branes and the D0-branes is understood through the centrifugal barrier. We discuss the implication on the stability of the D6+D0 solution.

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As it is well know there is a repulsive force between D6-branes and D0-branes both at short and at large distances [1]. On the other hand a solution corresponding to a background of D0-branes on D6-branes exist [2]. The background is stable classically, at least to quadratic order. Since super Yang-Mills (SYM) theory in seven dimensions is a non-renormalizable theory one cannot test the stability of the background at the quantum level. The situation would have been different if there had been an underline theory which flows at the IR to SYM. However, such a theory, which does not involve gravity, does not exist [3, 4, 5, 6]. In this short note we take advantage of the recent progress in the understanding of the relation between the near horizon geometry of a given branes configuration and the field theory living on the branes [7] and study the near horizon geometry of D6-branes with a constant field associated with the D0-brane. We consider the "decoupling" limit while keeping the super-Yang-Mills coupling constant and the field strength, associated with the D0-brane background, fixed. We find that the near horizon geometry is that of a spinning black hole in five dimensions. When the field strength is large (compared to $g_{YM}^{-4/3}$) the size of the black hole horizon is large (compared to the Planck scale) and hence the supergravity solution can be trusted in the analysis of the stability of the D0-branes background.

Before we consider the D6+D0 system let us review the decoupling limit of D6-branes. The "decoupling" limit is defined as follows [8]:

$$U = \frac{r}{\alpha^{'}} = \text{fixed}\,, \qquad g_{YM}^2 = (2\pi)^4 g_s \alpha^{'\,3/2} = \text{fixed}\,, \qquad \alpha^{'} \to 0\,. \tag{1}$$

In this limit we keep the field theory energies and coupling constant fixed while taking $\alpha^{'}$ to zero. This suggest that the D6-branes decouple from the bulk. However, as was noticed in [3, 4] in this limit $R_{11} \propto 1/\alpha^{'} \to \infty$, which means that the right description of the system is in M-theory as an ALE space with A_{N-1} singularity (where N is the number of D6-branes). Note that in this limit the Planck length is finite [3, 4]

$$l_p = (2\pi)^{-4/3} g_{YM}^{2/3}\,. \tag{2}$$

To be more precise we can start with the type IIA solution associated with D6-branes [9] and take the limit (1) to obtain [5]

$$\begin{aligned} ds^2 &= \alpha^{'}\left(\frac{(2\pi)^2}{g_{YM}}\sqrt{\frac{2U}{N}}dx_{||}^2 + \frac{g_{YM}}{(2\pi)^2}\sqrt{\frac{N}{2U}}dU^2 + \frac{g_{YM}}{(2\pi)^2\sqrt{2}}\sqrt{N}U^{3/2}d\Omega^2\right), \\ e^{\phi} &= \frac{g_{YM}^2}{2\pi}\left(2\frac{U}{g_{YM}^2 N}\right)^{3/4}. \end{aligned} \tag{3}$$

The solution can be trusted in the region $\frac{1}{g_{YM}^{2/3}N^{1/3}} \ll U \ll \frac{N}{g_{YM}^{2/3}}$ where both the curvature (in string units) and the effective string coupling are small [5].

For large U the effective string coupling becomes large and we need to uplift the solution to eleven dimensions to obtain

$$ds^2 = dx_{||}^2 + \frac{l_p^3 N}{2U}dU^2 + \frac{l_p^3 NU}{2}(d\tilde{\theta}^2 + \sin^2\theta d\varphi^2) + \frac{2Ul_p^3}{N}[d\phi + \frac{N}{2}(\cos\tilde{\theta} - 1)d\varphi]^2\,, \tag{4}$$

where $\phi \equiv x_{11}/R_{11}$ has period $\phi \sim \phi + 2\pi$. Defining the new variables

$$y^2 = 2Nl_p^3 U\,, \qquad \theta = \tilde{\theta}/2, \qquad \phi_1 = \varphi + \phi/N\,, \qquad \phi_2 = \phi/N\,, \tag{5}$$

we get the metric

$$ds^2 = dx_{||}^2 + dy^2 + y^2(d\theta^2 + \sin^2\theta d\phi_1^2 + \cos^2\theta d\phi_2^2)\,, \tag{6}$$

where $0 \leq \theta \leq \pi/2$ and $0 \leq \phi_1, \phi_2 \leq 2\pi$ with the identification $(\phi_1, \phi_2) \sim (\phi_1, \phi_2) + (2\pi/N, 2\pi/N)$. This identification leads to an ALE space with an A_{N-1} singularity.

Moreover, starting with near-extremal D6-branes with finite energy density above extremality we end up with a five dimensional Schwarzschild black hole sitting at the A_{N-1} singularity times R^6 [5]

$$ds^2 = -(1 - \frac{y_0^2}{y^2})dt^2 + \frac{dy^2}{(1 - \frac{y_0^2}{y^2})} + y^2 d\Omega_3^2 + dx_i^2\,, \tag{7}$$

where $i = 1, ..., 6$.

Now we wish to add D0-branes or in the field theory language we wish to find the supergravity solution associated with the D0-branes background of [2]. Namely, we keep the energy of the D0-branes background fixed while taking the limit (1). In M-theory D0-branes are described by gravitational waves along the $x_{10} - x_0$ direction. Thus they carry energy which contributes to the total mass of the black hole solution. Since they also carry momentum along x_{10} and since at the near horizon geometry of the D6-branes x_{10} is related to ϕ via $\phi = x_{10}/R_{10}$, the D0-branes will contribute also angular momentum to the black hole. This implies that the near horizon geometry of D6+D0 system is that of a spinning black hole. In fact, since from the 11D point of view D6-branes and D0-branes excite only the metric fields the solution is that of a chargeless black hole. Chargeless black holes in five dimensions are described by three parameters: the mass, the angular momentum in the, x_7, x_8 plane and the angular momentum in the x_9, x_{10} plan [10]. From eq.(5) it is clear that in our case

$$J_{7,8} = J_{9,10} \equiv J\,. \tag{8}$$

The solution is therefore [10]

$$\begin{aligned} ds^2 &= -dt^2 + \frac{(y^2+a^2)^3}{(y^2+2a^2)(y^2+a^2)^2 - \mu y^2} dy^2 + \\ &\quad + \frac{\mu y^2 (y^2+2a^2)}{(y^2+a^2)^3}(dt + a\sin^2\theta d\phi_1 + \cos^2\theta d\phi_2)^2 + \\ &\quad + (y^2+2a^2)(d\theta^2 + \sin^2\theta d\phi_1^2 + \cos^2\theta d\phi_2^2)\,. \end{aligned} \tag{9}$$

One way to check that this is indeed the near horizon geometry of D6+D0 branes is to use eq.(5) backwards and then reduce the solution along the ϕ direction to ten dimensions. Schematically (for more details see the appendix) one gets

$$ds^2 = A(U)(d\phi + \frac{N}{2}(\cos\theta - 1)d\varphi + B(U)dt)^2 + \tilde{ds}^2\,, \tag{10}$$

where $\tilde{ds}^2$ does not depend on ϕ and $d\phi$. We see, therefore, that an electric charge, associated with the D0-branes, appears and that the magnetic charge, associated with the D6-branes, is the same as in the solution with no D0-branes, as it should. Note that (8) is crucial to obtain 11D solution of the form (10).

The mass and angular momentum associated with this solution can be read from the asymptotic behavior of the solution and yields [10],

$$\begin{aligned} M &= \frac{3\pi\mu}{8G}\,, \\ J &= \frac{2}{3}Ma\,, \end{aligned} \tag{11}$$

where G is the five dimensional Newton constant. When the only source of energy is the D0-branes background (no additional thermal energy) the solution is extremal ($4a^2 = \mu$) and, hence, it does not Hawking radiates. This means that the D0-branes background is stable at the quantum level as well. To learn about the nature of this stability one can add some thermal noise on-top of the D0-branes background. By doing so one gets a non-extremal spinning five dimensional black hole ($4a^2 < \mu$). Such a black hole will Hawking radiates the angular momentum before it radiates the energy above extremality. From the field theory point of view this means that once we add some amount of energy on top of the D0-branes background the background becomes non-stable. This is, of course, in agreement with the fact that there is a repulsive force between the D0-branes and the D6-branes.

As we have seen, at the near horizon geometry of D6-branes the D0-branes contribute to the angular momentum. The repulsive force which they fill is, therefore, simply the centrifugal barrier, which in five dimensions has the form

$$V \propto \frac{L^2}{y^2} \propto \frac{L^2}{U}\,. \tag{12}$$

It is interesting to note that the same behavior was found in [11]. Note, however, that the regions of validity of the computations are different. In [11] there are two kinds of computations. The first uses the D-branes technique which is valid at the sub-stringy region. The second is based on the supergravity solution at large distances compared to the string scale (where the 1 in the harmonic function is kept). Our approach, in the spirit of [7], is valid at the sub-stringy region but it relies on supergravity. To trust the supergravity solution at the sub-stringy region we need the background field to be large, while to trust the sub-stringy computation of [11] one needs the background field to be small [5]. The fact that the potential is insensitive to the interpolation between the small and the large background field is in agreement with the result of branes probing for this configuration [12, 13, 14, 15].

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JHEP09(1998)018

Appendix

In this appendix we discuss in more details the relation between n_0, the number of D0-branes, and the five dimensional and the angular momentum of the black hole.

The type IIA solution of D6+D0 was presented in [13].[1] This solution is a simple generalization of the four dimensional solution of [18]. In the large U region, which corresponds to the large y region (where the mass and angular momentum of the black hole are defined), the gauge field part of the solution in the limit (1) is

$$A_\mu dx^\mu = \frac{\sqrt{3}QN}{8{\alpha'}^2U^2}dt + \frac{N}{2}(1-\cos\tilde{\theta})\delta\phi\,, \tag{13}$$

where

$$Q = \frac{g_s n_0 (2\pi)^6 {\alpha'}^{7/2}}{2V_6}\,. \tag{14}$$

Note that in this limit A_0 is proportional to $1/U^2$ and not to $1/U$. The reason is that we are in the large U region but not at the large r region. The full solution [13] in the large r region yields $A_0 \propto 1/r$.[2]

Since F_0 is held fixed while taking the limit (1) and since the number of D0-branes on the D6-branes is given by [2]

$$n_0 = \frac{1}{6(2\pi)^6}\int d^6x \mathrm{Tr} F\wedge F\wedge F\,, \tag{15}$$

n_0 is also fixed in this limit. This implies that $Q \propto {\alpha'}^2$ (where we have used eqs.(14), (1) and that A_0 is fixed , as expected.

Starting from the spinning black hole solution and reducing it to 10D one finds that

$$A_0 = \frac{\mu a}{y^4}. \tag{16}$$

Comparing to (13) one finds the right y dependence (since $y^2 \propto U$, eq. (5)). Moreover, eqs. (11), (13), (16) also verify that

$$J \propto n_0\,. \tag{17}$$

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JHEP09(1998)018